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The structure of D2-branes in the presence of an RR field

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Abstract: A Born-Infeld theory describing a D2-brane coupled to a 3-form RR potential is reconsidered and a new type of static solution is obtained which is even stable.

Keywords: D-branes, Solitons Monopoles and Instantons.

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1. Introduction

Recently in ref. [1] the question was considered whether a string can tunnel to a D2-brane in the presence of a uniform background RR field, and it was shown that the string can indeed nucleate the spheroidal bulge of a D2-brane and can tunnel to a toroidal D2-brane. The tunneling was described by bounces in euclidean time and the action of these entering the decay rate of the string into a toroidal D2-brane was deduced. The transition process was investigted in more detail in ref. [2], and the order of the quantum-classical transitions was determined depending on the magnitude of the applied RR field. All D2-branes and the euclidean time tunneling configurations (i.e. bounces) considered in these cases are unstable as stated in ref. [1], and the latter therefore represent saddle points. Here we are not concerned with the final states of such tunneling, but rather with the probability of tunneling away only as in ref. [3].

One can argue that depending on the potential, a euclidean time tunneling solution which is stable under small fluctuations in its neighbourhood can also exist as a configuration with finite action and hence as a local minimum of the action functional [4]. Our intention here is to search for such a configuration with finite energy. Since the transition rate exponent is proportional to minus the value of the configuration action (below we use instead the static energy), the tunneling via euclidean time branes with higher action is exponentially suppressed. Depending on the action of such other configuration, the qualitative tunneling consideration of ref. [1] then could or could not be made more quantitative within the approximations of the model.

The stability of Born-Infeld particles has been studied in detail in refs. [5, 6] in the case of the D3-brane (without an applied RR field). In particular it was shown there that the combined brane-antibrane configuration is unstable, whereas the Born-Infeld string is stable. The latter was also shown earlier in ref. [7], this being a consequence of the preservation of supersymmetry.

In ref. [1] it was shown that an initial state Born-Infeld string in the background of a uniform RR field is unstable and can tunnel via euclidean time saddle point brane configurations to some D2-brane. In the present work our original intention was to search for stable euclidean time D2-branes with minimal action which would therefore dominate other tunnelings. Instead we found a configuration of higher action than that of ref. [1], which, however, is stable under small fluctuations around it. This unusual property is worth observing.

In section 2 we consider other brane configurations and show that these are physically acceptable, i.e. are nonsingular and have finite energy (here we use the static solution in Minkowski time instead of the equivalent euclidean time configuration). We also show that for small values of the RR field there are two types of solutions, one type with lower action (static energy) (used in [1]) and a second, but stable type with higher action. In section 3 we consider special solutions. The equation for D2-brane configurations admits 3 types of solutions: Periodic solutions in terms of elliptic functions, two constant solutions of cylindrical shape, and solutions which are either finite or vanish exponentially at infinity. We are looking for branes with finite action, otherwise the tunneling rate would be zero. In particular we are considering the nucleation of the unwrapped string so that for $z \in \mathbb{R}^1$ space only the third type of solutions is physically acceptable. Periodic solutions can be used in the case of a compactified space, when the wrapped string tunnels into the toroidal D2-brane [2]. In section 4 we investigate the stability of such euclidean time tunneling branes by considering the fluctuation operator describing small deviations of the action in their vicinity, and demonstrate that in the case of our new configuration this has no negative eigenvalue. Vice versa it may possess a negative eigenvalue for the other type of solution and imply that such a configuration is a saddle point. In section 5 we conclude with some remarks.

2. Formulation of the problem and conditions for static solutions with finite energy

With the convention of $\alpha' = 1$ the action of a D2-brane coupled with the 3-form gauge potential A in Born-Infeld approximation is given by [1, 8]

$$I = -\frac{1}{4\pi^2 g}\int d^3\xi \left\{ \sqrt{-\det\left(g^{\rm ind}_{\alpha\beta} + 2\pi\mathcal{F}_{\alpha\beta}\right)} + \frac{1}{3!}\epsilon^{\alpha\beta\gamma}A_{\mu\nu\rho}\partial_\alpha X^\mu \partial_\beta X^\nu \partial_\gamma X^\rho \right\}, \tag{2.1}$$

where $\mu,\nu,\rho = 0,\ldots,9$ are spacetime indices, and $\alpha,\beta,\gamma = 0,1,2$ worldvolume indices and g is the string coupling. The dilaton field is taken to be constant and the background field strength $H = dA$ is taken to be uniform and aligned with the brane, i.e. $H_{0123} = h = \text{const.}$ As in ref. [1] we choose the world volume to be cylindrical and hence define

$$X^0 = t\,, \qquad X^1 = z\,, \qquad X^2 = R(t,z)\cos\sigma\,, \qquad X^3 = R(t,z)\sin\sigma\,, \qquad \mathcal{E} = 2\pi F_{tz}\,, \tag{2.2}$$

and all other $X^i = \text{const.}$ After integration over σ, the action takes the form

$$I = \int dt \int dz \mathcal{L}\,, \qquad \mathcal{L} = -\frac{1}{2\pi g}\left(R\sqrt{1 - \dot{R}^2 - \mathcal{E}^2 + R'^2} - \frac{h}{2}R^2\right), \tag{2.3}$$

where dots and primes denote derivatives with respect to t and z respectively. We observe that the 1-time-1-space combination $-\dot{R}^2 + R'^2$ is similar to that of $1+1$ dimensional soliton theory, in which the soliton of the static theory is identical with the instanton of the euclideanised 1-dimensional theory. In the following we shall make use of this correspondence and consider instead of the action the energy of the static configuration of the 2-dimensional theory. The time dependence of the latter can be restored by Lorentz boosting. The canonical momentum $D = 2\pi g \delta I/\delta E$ must be constant (cf. refs. [1, 2]) and is given by gn, where n is the number of fundamental strings (cf. [1, 2]), each of tension $1/2\pi$. For static solutions the energy E is given by

$$E = \frac{1}{2\pi g}\int dz \left\{ \sqrt{(1+R'^2)(D^2+R^2)} - \frac{h}{2}R^2 \right\}. \tag{2.4}$$

Using the static expression (2.4) instead of the proper action (2.3) for euclidean time in the computation of the decay rate, one has to subtract from the action that of n fundamental strings as emphasized in ref. [1].

Variation of E with respect to R yields

$$\frac{\delta E}{\delta R} - \frac{d}{dz}\frac{\delta E}{\delta R'} = 0\,, \tag{2.5}$$

which can be reduced to a first order differential equation because it does not contain the variable z explicitly, resulting in

$$C = \sqrt{\frac{R^2+D^2}{1+R'^2}} - \frac{h}{2}R^2\,, \tag{2.6}$$

where C is a constant. We rewrite this equation

$$R' = \mp \frac{h}{hR^2+2C}\sqrt{(R_+^2 - R^2)(R^2 - R_-^2)}\,, \qquad R' \neq 0 \tag{2.7}$$

with

$$R_\pm{}^2 = \frac{2}{h^2}\left[(1-Ch) \pm \sqrt{1-2Ch+h^2D^2}\right]. \tag{2.8}$$

We observe that for physical reasons (R has to be real)

$$R_-{}^2 \leq R^2 \leq R^2{}_+$$

which implies motion between these bounds, i.e. periodic motion. Now we compare the expression (2.6) for the integration constant C with expression (2.4) for the energy density. We observe that the second (i.e. "potential") terms are the same and the first ("kinetic") terms differ by a multiplicative factor, i.e. $(1+R'^2)$, and consequently one may be led to believe that positive energy requires positive C. Here we abandon this expectation and consider negative values of C. At first sight this may seem dangerous since this suggests a pole in eq. (2.7) but in spite of this the solution is nonsingular and the energy finite as will be shown below. In this case eq. (2.7) permits solutions for all values of h and for R_+^2 and R_-^2 real and positive. From eq. (2.8) we deduce two conditions

$$C^2 \geq D^2\,, \qquad 1 - Ch \geq 0\,,$$

which for positive values of C imply

$$hD \leq hC \leq 1$$

meaning that h must be less than the critical value $h_C = 1/D$. On the contrary, for negative values of C, keeping in mind that h is positive, we have only the one condition that

$$|C| \geq D$$

(which excludes $R^2_{\min} < 0$) so that h can take any value, but $|C|$ is restricted to non-small values. At the end of the next Section we present a solution for large values of h, i.e. when $hD \gg 1$.

We now demonstrate that solutions for negative values of C are, in principle, also acceptable, i.e. are associated with a finite energy. It is convenient in this case to set

$$C = -\frac{h}{2}a^2 \,. \tag{2.9}$$

Then for $|C| \geq D$ we have

$$0 \leq R_-^2 \leq a^2 \leq R_+^2$$

and

$$R' = \mp \frac{\sqrt{(R_+^2 - R^2)(R^2 - R_-^2)}}{R^2 - a^2} \,.$$

We consider the approach $R \to a$. In this domain

$$R' \simeq \mp \frac{\lambda}{2}\frac{1}{R-a} \,, \qquad \lambda \simeq \frac{2\sqrt{a^2+D^2}}{ha}$$

and, with integration constant z_1,

$$(R-a)^2 = \lambda |z - z_1| \,, \qquad |R'| = \frac{1}{2}\sqrt{\frac{\lambda}{|z-z_1|}}$$

so that the crucial part of the energy (2.4) around $z = z_1$ becomes

$$\begin{aligned} \frac{1}{2\pi g}\int_{z_1-\delta}^{z_1+\delta} dz \sqrt{(1+R'^2)(D^2+R^2)} &\simeq \frac{\sqrt{a^2+D^2}}{2\pi g}\int_{z_1-\delta}^{z_1+\delta} |R'| dz \\ &= \frac{\sqrt{\lambda(a^2+D^2)}}{4\pi g}\int_{z_1-\delta}^{z_1+\delta} \frac{dz}{|z-z_1|^{1/2}} < \infty \,. \end{aligned} \tag{2.10}$$

We conclude therefore that static solutions for negative values of C also have finite energy.

3. Solutions with wheel-like shape

We now consider some special solutions and their energy. We consider first the special case $C = -D$ (i.e. $C < D$). In this case $D = ha^2/2$, $R_- = 0$, $R_+^2 = 4(1+hD)/h^2$. Integrating the equation

$$R' = \mp \frac{R}{R^2 - a^2}\sqrt{R_+^2 - R^2}$$

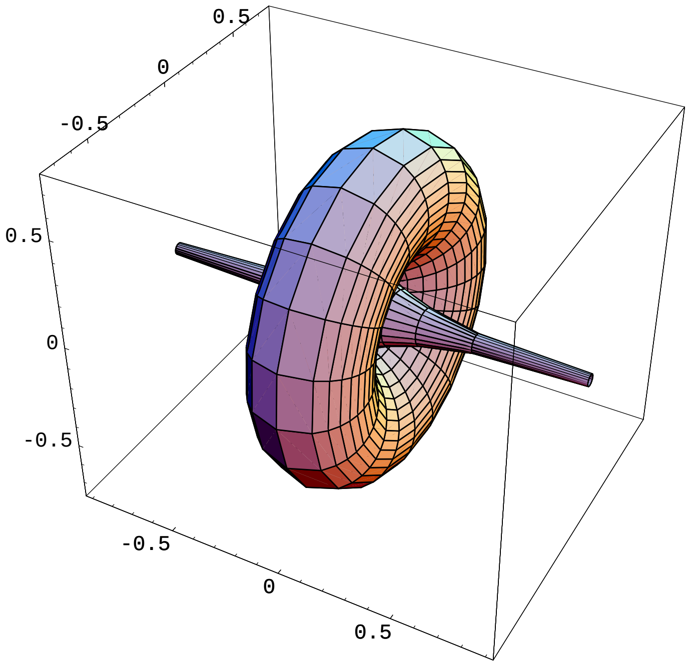

Figure 1: The solution for a negativa value of C. $C = -D$, $D = 0.4$, $h = 4$.

one obtains (note that $z = z_0$ corresponds to $R = R_+$, $z = +\infty$ to $R = 0$ for the lower sign and $z = -\infty$ to $R = 0$ for the upper sign)

$$\mp(z - z_0) = -\sqrt{R_+^2 - R^2} + \frac{a^2}{R_+} \ln \frac{R_+ + \sqrt{R_+^2 - R^2}}{R} \tag{3.1}$$

for $z - z_0 \lessgtr 0$ respectively. In figure 1 we show the combination of these two solutions, one representing the continuation (or mirror image) of the other across the R-axis. One can clearly see that the effect of negative values of C is opposite to that of positive C: Whereas positive C yield an elongated spheroidal bulge (cf. refs. [1, 2]), negative C push this bulging in the opposite direction (thereby crossing the R-axis at a point $R = R_0$ close to the origin) to eventually form a wheel-like structure. We calculate the energy of this configuration using eq. (2.6) with C there replaced by $-D$. Substituting in the expression (2.4) for the energy the expression for $(1 + R'^2)$ obtained from eq. (2.6) one obtains

$$E = \frac{1}{2\pi g} \int dz \left\{ \frac{\pm(D^2 + R^2)}{C + \frac{h}{2}R^2} - \frac{h}{2}R^2 \right\}.$$

Here we have to choose the minus sign in order to obtain a positive expression (even for the divergent part, cf. the comment after eq. (2.4)). With this choice and rewriting the expression slightly, one has

$$E = \frac{1}{2\pi g} \int dz \left\{ \frac{D(\frac{h}{2}R^2 - D) - (R^2 + \frac{h^2}{4}R^4)}{\frac{h}{2}R^2 - D} \right\}.$$

This expression can now be rewritten as

$$E = \frac{D}{2\pi g}\int dz - \frac{1}{4\pi gh}\int_{-\infty}^{\infty} dz \frac{R^2(4+h^2R^2)}{(R^2-a^2)} = \frac{D}{2\pi g}\int dz + 2E_- ,$$

where

$$E_- = \frac{1}{4\pi gh}\int_0^{R_+} dR \frac{dz}{dR}\frac{R^2(4+h^2R^2)}{(R^2-a^2)} .$$

In this expression we now have to choose the solution with the sign as in

$$R' = \frac{R\sqrt{R_+^2 - R^2}}{R^2 - a^2}$$

(as in ref. [1] this ensures that $R \to 0$ for $z \to \infty$). With $D = +ng$ one now obtains

$$E = \frac{n}{2\pi}\int dz + \frac{2R_+}{\pi gh} + \frac{2h}{8\pi^2 g}\left(\frac{4\pi}{3}R_+^3\right), \tag{3.2}$$

whereas the energy of the spheroidal bulge of ref. [1] is

$$E = \frac{n}{2\pi}\int dz + \frac{h}{8\pi^2 g}\left(\frac{4\pi}{3}R_+^3\right).$$

We observe that the energy of the bulge (the term with R_+^3 which results from $R' \neq 0$) is here twice that of the case considered in ref. [1], in addition to another term. Thus our wheel-like solution has a higher energy.

Next we consider the case $C = -ha^2/2$ with $ha \gg 1, hD \gg 1$. In this case

$$R_\pm^2 \simeq a^2 \pm 2R_0 , \qquad R_0 \equiv \sqrt{a^2 + D^2}/h , \qquad R_0 \ll a^2, D^2$$

We set (with $\tau' \equiv d\tau/dz$)

$$R^2 \equiv a^2 + \tau .$$

Then eq. (2.7) becomes

$$\frac{\tau'}{2\sqrt{a^2+\tau}} = \mp\frac{\sqrt{4R_0^2 - \tau^2}}{\tau} . \tag{3.3}$$

Integration (for $\tau \ll a^2$) yields the equation ($R^2 = x^2 + y^2$)

$$z^2 + \left(\frac{R^2 - a^2}{2a}\right)^2 = \frac{a^2 + D^2}{h^2a^2} \tag{3.4}$$

(with integration constant $z_0 = 0$). This equation describes a deformed circular structure with radius a (neglecting $2\sqrt{a^2+D^2}/h$) which is obvious if we look at its intersection with the plane $z = 0$. The structure has a thickness $\sqrt{a^2+D^2}/ha$. Thus in the limit $ha \gg 1$ with parameter a fixed, the circular structure becomes a shell of finite radius and small thickness as shown in figure 2. A configuration like this has been observed computationally in ref. [9] for a condition similar to the one here, i.e. $C < 0$.

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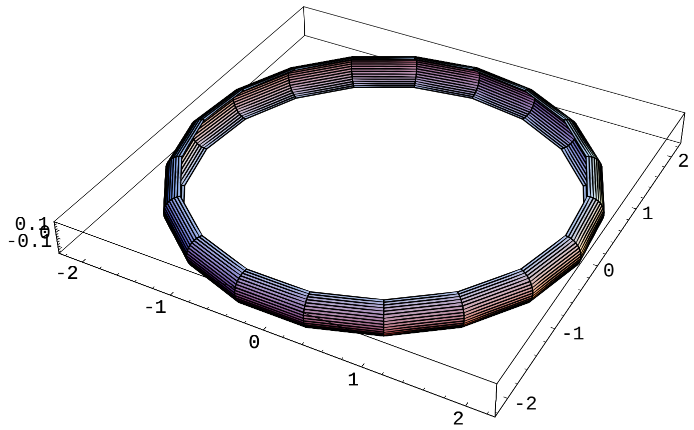

Figure 2: The limit of the string becoming an annular shell $a = 2$, $D = 3$, $h = 10$.

4. The stability of the solutions

We now return to the question of whether the euclidean time tunneling solutions considered above are stable, i.e. are global minima of the action, here considered as energy, or not. For this reason we consider the second variation $\delta^2 E$ of the energy in the vicinity of the classical solution. Straightforward calculation yields

$$\delta^2 E = \frac{1}{4\pi g} \int \delta R \hat{M} \delta R dz \,, \tag{4.1}$$

where

$$\hat{M} = -\frac{d}{dz} Q \frac{d}{dz} + 2P \frac{d}{dz} + V \tag{4.2}$$

with

$$Q = \frac{\sqrt{R^2 + D^2}}{(1 + R'^2)^{3/2}} \,, \qquad P = \frac{RR'}{\sqrt{(1 + R'^2)(R^2 + D^2)}} \,, \qquad V = D^2 \frac{\sqrt{1 + R'^2}}{(R^2 + D^2)^{3/2}} - h \,. \tag{4.3}$$

A negative eigenvalue of the fluctuation operator $\hat{M}$ implies instability of the respective solution, since the variation of the solution in the direction of the corresponding eigenfunction decreases the energy. Therefore it suffices to investigate for which solutions $\hat{M}$ has only positive eigenvalues and for which not.

One can derive another expression for $\hat{M}$ which is equivalent to the one above, i.e.

$$\hat{M} = -\frac{1}{R'} \frac{d}{dz} R'^2 Q \frac{d}{dz} \frac{1}{R'} + 2P \frac{d}{dz} - \frac{1}{R'} (QR'')' \tag{4.4}$$

but allows to present $\delta^2 E$ as a sum of positively defined terms plus a term proportional to C. On the basis of this one can easily distinguish the stable solutions from the unstable ones.

Here we are looking for branes with finite energy. With $z \in \mathbb{R}^1$ this implies the square integrability of the eigenfunctions of $\hat{M}$. The charge D remains fixed by quantisation (as stated earlier). The positivity of all eigenvalues of $\hat{M}$ means that the mean value of $\hat{M}$

over any function with finite norm is positive and vice versa. We assume that $\delta R(z)$ is a square integrable function, i.e.

$$\int \delta R(z)^2 dz < +\infty\,, \tag{4.5}$$

and consider the mean value of $\hat{M}$ on that class of functions. This is the same as $\delta^2 E$. The function $R(z)$ is bounded, $R_- \leq R \leq R_+$, and $\delta R(z)$ must vanish at infinity. Consequently we can integrate the term $2\delta R P \frac{d}{dz}\delta R = P\frac{d}{dz}\delta R^2$ by parts and the total derivative must vanish. Also in the first term we can return to the antihermitean operator d/dz to act on the term to the left yielding a minus sign. As a result we have

$$\delta^2 E = \frac{1}{4\pi g}\int \left[R'^2 Q\left(\frac{d}{dz}\frac{\delta R}{R'}\right)^2 + U\delta R^2\right] dz\,, \tag{4.6}$$

where

$$U = V - \frac{dP}{dz} - \frac{1}{R'}\left(QR''\right). \tag{4.7}$$

It is worth noting that one can make analogous manipulations with the differential equation for eigenvalues of the operator $\hat{M}$. After appropriate substitutions and eliminating the first order derivative, one obtains the same expression (4.7) for the effective potential. We prefer the above way which allows us to connect the second variation of the energy directly with the integration constant C. With some algebra one can deduce the explicit expression for U:

$$U = h\frac{R^2}{R^2+D^2} + hD^2\frac{R^2}{(R^2+D^2)^2} - 2C\frac{R^2}{(R^2+D^2)^2}\,. \tag{4.8}$$

The last term demonstrates in a transparent way the stability of solutions with negative values of C. In contrast, in the case when $C > 0$ and $hD^2 \leq C$, the negative term becomes consequently a source for the instability which is due to negative eigenvalues.

5. Conclusions

In the above we obtained a new nonsingular, finite-energy (or rather action) solution of the Born-Infeld theory of a D2-brane in the presence of a three-form RR-potential. The wheel-like shape of this solution appears as a natural consequence of negative values of the constant C which imply a threading of the fundamental strings through its bulging in a direction which is opposite to that of the spheroidal structure of ref. [1]. Also naturally one expects the more complicated structure found here to have a higher energy. By considering small fluctuations about a particular wheel-shaped configuration we also demonstrated its stability.

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References

[1] R. Emparan, *Born-Infeld strings tunneling to D-branes*, *Phys. Lett.* **B 423** (1998) 71 [hep-th/9711106].

[2] D.K. Park, S. Tamarian, Y.G. Miao and H.J.W. Müller-Kirsten, *Tunneling of Born-Infeld strings to D2-branes*, *Nucl. Phys.* **B 606** (2001) 84 [hep-th/0011116].

[3] E.B. Bogomolnyi and V.A. Fateyev, *Phys. Lett.* **B 71** (1977) 93.

[4] G.W. Gibbons, K. Hori and P. Yi, *String fluid from unstable D-branes*, *Nucl. Phys.* **B 596** (2001) 136 [hep-th/0009061].

[5] D.K. Park, S.N. Tamarian, H.J.W. Müller-Kirsten and J.-Z. Zhang, *D-branes and their absorptivity in Born-Infeld theory*, *Nucl. Phys.* **B 594** (2001) 243 [hep-th/0005165].

[6] G.W. Gibbons, *Born-Infeld particles and Dirichlet p-branes*, *Nucl. Phys.* **B 514** (1998) 603 [hep-th/9709027].

[7] C.G. Callan and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, *Nucl. Phys.* **B 513** (1998) 198 [hep-th/9708147].

[8] M. Li, *Boundary states of D-branes and Dy-strings*, *Nucl. Phys.* **B 460** (1996) 351 [hep-th/9510161];
M.R. Douglas, *Branes within branes*, hep-th/9512077;
M. Green, J.A. Harvey and G. Moore, *1-brane inflow and anomalous couplings on D-branes*, *Class. and Quant. Grav.* **14** (1997) 47 [hep-th/9605033];
J. Pawełczyk and S.-J. Rey, *Ramond-Ramond flux stabilization of D-branes*, *Phys. Lett.* **B 493** (2000) 395 [hep-th/0007154].

[9] Y. Hyakutake, *Torus-like dielectric D2-brane*, *J. High Energy Phys.* **05** (2001) 013 [hep-th/0103146].

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